Fundamental Concepts for Environmental Processes
Problem Sets With Solutions

Overview of Water Quality

A sample of water was analyzed for solids content. 0.25L of sample was filtered through a 2g filter paper. 10% of the filtrate was placed in a crucible that weighed 30g. After drying, the total weight of the filter and crucible are 2.15g and 30.050g, respectively. After ashing, the filter paper and crucible weigh 2.05g and 30.009g, respectively. Calculate the solids concentrations of the following:

a) TSS and VSS
b) TDS and VDS

$$\text{TSS} = \frac{2.15 - 2.00}{0.25} = *1000 = 600\,\tfrac{mg}{L}$$

$$\text{VSS} = \frac{2.15 - 2.05}{0.25} *1000 = 400\,\tfrac{mg}{L}$$

$$\text{TDS} = \frac{(30.050 - 30.000)}{0.025} *1000 = 2000\,\tfrac{mg}{L}$$

$$\text{VDS} = \frac{(30.050 - 30.009)}{0.025} *1000 = 1640\,\tfrac{mg}{L}$$

A 0.15 L sample of water has an initial dissolved oxygen level of 8.35 mg/L. After five days in incubation in a 300 mL BOD bottle, the dissolved oxygen level has decreased to 4.09 mg/L. Calculate the BOD_5.

$$BOD_5 = \frac{(8.35 - 4.09)}{\frac{150}{300}} = 8.52\,\tfrac{mg}{L}$$

A small industrial plant discharges wastewater into a stream. The plant discharge has a flow rate of 0.2 m^3/s and a BOD_u concentration of 40 mg/L. The stream receiving the discharge has a flow rate of 1.3 m^3/s and a BOD_u of 6 mg/L. Downstream of the discharge point, the velocity in the stream is 0.15 m/s. Assuming no other sources of BOD enter the stream, what is the BOD_u remaining in the stream 10 km downstream of the plant discharge? Use k = 0.23 1/day.

$$\dot{m}_{\text{BOD p}} = 0.2 \frac{m^3}{s} * 40 \frac{g}{m^3} = 8 \frac{g}{s}$$

$$\dot{m}_{\text{BOD p}} = 1.3 \frac{m^3}{s} * 6 \frac{g}{m^3} = 7.8 \frac{g}{s}$$

$$\text{BOD} = \frac{(8+7.8)}{(0.2+1.3)} = 10.533 \frac{mg}{L}$$

$$t = \frac{(10*1000)m}{0.15\frac{m}{s}} = 66667\text{s} = 18.52 \text{ hr} = 0.7716 \text{ d}$$

$$\text{BOD}_t = 10.533\ (1 - e^{-0.23*0.7716}) = 1.713 \frac{mg}{L}$$

Fluid Mechanics

A mixing tank receives wastewater from two inlet lines. The first inlet has an i.d. of 8 in and a flow velocity of 2 ft/s. The second inlet has an i.d. of 12 in and a flow velocity of 2.5 ft/s. The mixed water leaves the tank through a 16 in i.d. pipe. What is the flow velocity in the outlet pipe?

$$q_1 = 2 * \frac{\pi}{4} \left(\frac{8}{12}\right)^2 = 0.6981\text{cfs}$$

$$q_2 = 2.5 * \frac{\pi}{4} \left(\frac{12}{12}\right)^2 = 1.9635\text{cfs}$$

$$v_{out} = \frac{2.6616}{\frac{\pi}{4}\left(\frac{16}{12}\right)^2} = 1.906 \frac{ft}{s}$$

Flow is carried in a triangular channel at a velocity of 2 ft/s. The channel is 2 ft deep and has a top width of 12 ft. The flow in the channel is 1 ft deep. What is the flow rate in the channel?

$$q = \tfrac{1}{2}\,(6 * 1) * 2$$
$$= 6 \text{ cfs}$$

A pipe flowing full carries water at 20°C from a reservoir in 12 inch ID pipe at a rate of 3 ft/s. Some of the flow branches off to a cooling system while the remaining flow continues on for treatment in 10 inch ID pipe at 2 ft/s. What diameter pipe (inches) is necessary to deliver water to the cooling system at 1 ft/s?

$$q_{in} = 3 * \frac{\pi}{4}\left(\frac{12}{12}\right)^2 = 2.3562 \text{ cfs}$$

$$q_{out} = 2 * \frac{\pi}{4}\left(\frac{10}{12}\right)^2 = 1.0908 \text{ cfs}$$

$$A_{cool} = \frac{1.2654}{1\frac{ft}{s}} = 1.2654 \text{ ft}^2$$

$$d = 1.269 \text{ ft} = 15.23 \text{ in}$$

Two triangular streams merge to form one larger channel with a rectangular cross-section. The diagram showing the characteristics of the streams follows, where w is stream width, d is flow depth, and v is flow velocity:

Determine the flow velocity in stream 2.

$$q_1 = \frac{1}{2}(2 * 0.5) * 1.5 = 0.75 \frac{m^3}{s}$$

$$q_3 = (2 * 0.75) * 1.5 = 2.25 \frac{m^3}{s}$$

$$v_2 = \frac{2.25 - 0.75}{\frac{1}{2}(3 * 0.5)} = 2.00 \frac{m}{s}$$

Conservation of Mass

A stream with a 50 mg/L concentration of some chemical flows at 0.75 m^3/s. What is the mass flow rate in g/s of the chemical?

$$\dot{m} = 50 * 0.75 = 37.5 \frac{g}{s}$$

The oxygen concentration in a 2L bottle of water is 5.0 mg/L. What is the massing of O_2 in the bottle?

$$\dot{m} = 5.0 * 2 = 10mg = 0.01g$$

Wet sludge with a solids content of 30% by weight is placed in a drying bed until it reaches a solids content of 80%. If 5 tons of wet sludge are placed in the bed:

a) how much dried sludge is removed, and
b) how much water (by weight) is lost?

$$m_{solids} = 5 * 2000 * 0.30 = 3000 \text{ lb}$$

$$m_{dry-sludge} = \frac{3000}{0.8} = 3750 \text{ lb}$$

$$m_{water-lost} = 10000 - 3750 = 6250 \text{ lb}$$

Water with a solids concentration of 50 mg/L passes through a sand filter at a flow rate of 2 m^3/s. Effluent from the filter has a solids concentration of 8 mg/L. At what rate do solids accumulate on the sand filter?

$$\dot{m}_{solids-in} = 50 * 2 = 100 \frac{g}{s}$$

$$\dot{m}_{solids-out} = 8 * 2\, 16 \frac{g}{s}$$

$$\dot{m}_{solids-acc.} = 100 - 16 = 84 \frac{g}{s}$$

Two pipes, Pipe A and Pipe B, join and flow into Pipe C. Pipe A has a flow of 0.1 m^3/s and a concentration of 5 mg/L and Pipe B has a flow of 0.3 m^3/s and a concentration of 9 mg/L. What is the concentration of the flow in Pipe C?

$$\dot{m}_A = 5 * 0.1 = 0.5 \frac{g}{s}$$

$$\dot{m}_B = 0.3 * 9 = 2.7 \frac{g}{s}$$

$$\dot{m}_C = 0.5 + 2.7 = 3.2 \frac{g}{s}$$

$$C_C = \frac{3.2 \frac{g}{s}}{(0.1 + 0.3)} = 8 \frac{mg}{l}$$

Many larger wastewater treatment plants use activated sludge systems to remove organic material from wastewater. This process generates lots of solids that then must be removed in clarifiers. In one system, a flow of 500 m^3/day enters a clarifier with a solids concentration of 10,000 mg/L. Of the water entering the clarifier, 90 % leaves as treated effluent. That effluent carries 0.1 % of the solids. The remaining water and solids leave the clarifier as waste sludge. Determine:

a) What is the solids concentration in the treated effluent?

b) How many kilograms of dry solids exit the system in waste sludge every day?

Mass balance of water:
Water entering system = 500 m^3/day
Water leaving as treated effluent = 0.90 * 500 = 450 m^3/day
Water leaving with sludge = 500 - 450 = 50 m^3/day

Mass balance of solids:
Solids entering system = 500 * 10,000 = 5,000,000 g/day
Solids exiting system with treated effluent = 0.001 * 5,000,000 = 5,000 g/day
Solids leaving with sludge = 5,000,000 - 5000 = 4,995,000 g/day = 4995 kg/day

Solids concentration in effluent = 5000/450 = 11.11 mg/L

Water entering a septic tank has a maximum flow of 0.025 m^3/min, a concentration of solids of 200 mg/L, and a concentration of organic material of 150 mg/L. Approximately 60 % of the solids and 30 % of the organics are retained in the septic tank.

a) What are the mass rates of solids and organic material exiting the tank?
b) What are the effluent concentrations of solids and organic material?

$$\dot{m}_{\text{solids-in}} = 0.025 * 200 = 5\,\frac{\text{g}}{\text{m}}$$

$$\dot{m}_{\text{org-in}} = 0.025 * 150 = 3.75\,\frac{\text{g}}{\text{m}}$$

$$\dot{m}_{\text{solids-out}} = 5 * (1 - .6) = 2\,\frac{\text{g}}{\text{m}}$$

$$\dot{m}_{\text{BOD-out}} = 3.75\,(1 - .3) = 2.625\,\frac{\text{g}}{\text{m}}$$

$$C_{\text{solids-out}} = \frac{2}{0.025} = 80\,\frac{\text{mg}}{\text{l}}$$

$$C_{\text{orgs-out}} = \frac{2.625}{0.025} = 105\,\frac{\text{mg}}{\text{l}}$$

Water enters a settling pond from two rectangular channels. The first influent flow has a depth of 0.2 m, a width of 1 m, a velocity of 0.5 m/s, and a concentration of 500 mg/L. The second influent flow has a depth of 0.15 m, a width of 1.5 m, a velocity of 0.75 m/s, and a concentration of 400 mg/L. A single rectangular channel carries effluent from the pond. If 95% of the incoming solids are retained in the pond, what is the solids concentration of the effluent?

$$\dot{m}_1 = (0.2 * 1)(0.5)(500) = 50\,\frac{g}{s}$$

$$\dot{m}_2 = (0.15 * 1.5)(0.75)(400) = 67.5\,\frac{g}{s}$$

$$\dot{m}_{006} = 0.05\,(50 + 67.5)5.875\,\frac{g}{s}$$

$$C = \frac{5.875}{(.2 * 1)(.5) + (.15 * 1.5)(.75)} = 21.86\,\frac{mg}{l}$$

First Order Reactions

Calculate the hydraulic residence time for a plug flow reactor that has a volume of 1150 gal and a flow rate in of 25 gal/hour.

$$\theta = \frac{1150}{25} = 46\,\mathrm{hr}$$

A batch reactor is used to break down organic materials in a wastewater. The initial concentration is 500 mg/L and the reaction rate is 0.5 1/d. What is the concentration after:

a) 1 day, and
b) 1 week?

$$C_1 = 500\,e^{-0.5(1)} = 303.3\,\frac{mg}{l}$$

$$C_7 = 500\,e^{-0.5(7)} = 15.10\,\frac{mg}{l}$$

What size (volume) tank would you need to treat waste water at a flow rate of 0.30 m^3/hr from a concentration of 1.75 mg/L to a final concentration of 0.75 mg/L? k = 0.25l/d. Do for both CSTR and Plug Flow reactors.

CSTR

$$0.75 = \frac{1.75}{1 + 0.250}$$

$$\theta = 5.33\,\mathrm{d}\ \cancel{v} = .30\,\frac{m^3}{hr} * 5.33 * 24 = 38.4 m^3$$

PFR

$$0.75 = 1.75e^{-025\theta}$$

$$\theta = 3.389\mathrm{d}\ \cancel{v} = 0.30 * 3.389 * 24 = 24.40\,m^3$$

Wastewater flow from a food processing operation is passed through two reactors in series (i.e. all flow passes through the first reactor then the second). The reactors are used to reduce the concentration of Constituent X, which has an initial concentration of 1000 mg/L. A constant flow of 80 ft^3/hr is processed by the reactors. The first reactor has a volume of 4000 ft^3 and is equipped with an agitator to keep the fluid well-mixed. The second reactor has a volume of 6000 ft^3 and provides no mixing of fluids. If the reaction rate coefficient for Constituent X is 0.5 1/day, what is the concentration of Constituent X in the effluent of the second reactor?

$$\theta_1 = \frac{4000}{80} = 50\,hr$$

$$C_1 = \frac{1000}{1 + 0.5\left(\frac{50}{24}\right)} = 489.8\,\frac{mg}{l}$$

$$\theta_2 = \frac{6000}{80} = 75\,hr = 3.125\,d$$

$$C_2 = 489.8\,e^{-0.5(3.125)} = 102.67\,\frac{mg}{l}$$

Sedimentation

Calculate the diameter and depth (in m) of a circular sedimentation basin for a design flow of 3800 m^3/d based on an critical velocity of 0.00024 m/s and a detention time of 3 hours.

$$\text{Vol} = 3800 * \frac{3}{24} = 475\ \text{m}^3$$

$$A = \frac{3800}{0.00024 * 24 * 3600} = 183\ \text{m}^2$$

$$\text{depth} = 2.6\ \text{m} \qquad\qquad d = 15.28\text{m}$$

Water entering a wastewater treatment plant is passed through a grit chamber. The flow rate is 0.12 m^3/s and the chamber surface area is 48 m^2. What is the smallest diameter particle (give particle diameter in mm) that will be completely retained in the grit chamber? Assume the particles are mineral and that the water temperature is 15 °C.

$$V_s = \frac{0.12}{48} = 0.0025\ \frac{\text{m}}{\text{s}}$$

$$0.0025 = \frac{(2650 - 999)\ d^2\ 9.81}{18 * 1.14\ \text{x}\ 10^{-3}}$$

$$d = 0.056\ \text{mm}$$

Two pipes carry mineral solids having a particle size distribution as shown below into a grit chamber. Pipe 1 has an inside diameter of 0.20 m, a flow velocity of 0.6 m/s, and a solids concentration of 400 mg/L. Pipe 2 has an inside diameter of 0.10 m, a flow velocity of 0.8 m/s, and a solids concentration of 700 mg/L. If the tank has a surface area of 2.8 m^2, what is the approximate accumulation (storage) rate of solids in the tank as kg/hr? Assume a water temperature of 20 C.

Particle Size Range, mm	% Composition
> 2.00	10
1.00 - 2.00	14
0.50 - 1.00	27
0.25 - 0.50	28
0.10 - 0.25	13
< 0.10	8

$$\dot{m}_1 = 0.0188 * 400 = 7.5398 \frac{g}{s}$$

$$q_2 = 0.8 * \frac{\pi}{4} (0.1)^2 = 0.0063 \frac{m^3}{s}$$

$$\dot{m}_2 = 700 * 0.0063 = 4.398 \frac{g}{s}$$

$$\dot{m}_{tot} = 11.938 \frac{g}{s}$$

$$q_{tot} = 0.0188 + 0.0063 = 0.251 \frac{m^3}{s}$$

$$V_c = \frac{0.0251}{2.8} = 0.009 \frac{m}{s}$$

$$d_c = \frac{(2650 - 998) d^2 9.81}{18 * 1x10^{-3}} = 0.009$$

$d_c = 0.10mm$

92% captured

$$\dot{m}_{acc} = 11.938 * 0.92 = 10.98 \frac{g}{s}$$